Title: Logarithmic Equations, Level I
Class: Math 107 or Math 111
Author: Lindsey Bramlett-Smith
Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: logarithmic equations, equations with logarithms, solving logarithmic equations, solving logarithm equations

Logarithmic Equations, Level I

Purpose: This is intended to refresh your skills in solving logarithmic equations.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

- **Definition:** for $b > 0$, $b \neq 1$, $log_b\, a = x$ is equivalent to $b^x = a$.

  the answer to the logarithm is the exponent

  Note that the base *b* is a positive number, and that the number you are taking the logarithm of, *a*, is also a positive number. But, the answer to the logarithm, *x*, may be a negative number.

- **Solve** logarithmic equations that have the form $log_b\, a = x$ by converting into an exponential equation that has the form $b^x = a$.

Example 1

$$log_9\, x = \tfrac{3}{2}$$
$$9^{3/2} = x \quad \text{Converted the logarithm to an exponential}$$
$$\left(\sqrt{9}\right)^3 = x$$
$$(3)^3 = x$$
$$27 = x$$

Practice 1 $log_x\, 25 = 2$

Did you get *x* = 5? Did you also get *x* = -5, but reject it since we can't have negative bases?

- **Solve** logarithmic equations that are more complicated by using the properties of logarithms to rewrite the equation so that it contains just one logarithm.

Properties of Logarithms

1) $log_b\, M + log_b\, N = log_b\,(MN)$

2) $log_b\, M - log_b\, N = log_b\left(\frac{M}{N}\right)$

3) $log_b\, M^r = r\, log_b\, M$

and $log\, M = log_{10}\, M$ & $ln\, M = log_e\, M$

Logarithmic Forms that can NOT be rewritten

$log_b\,(M+N)$ **nor** $(log_b\, M)(log_b\, N)$

$log_b\,(M-N)$ **nor** $\dfrac{log_b\, M}{log_b\, N}$ (except as a change of base)

$(log_b\, M)^r$

Example 2 $\log x = 1 - \log(x-3)$

$\log x + \log(x-3) = 1$

$\log[x(x-3)] = 1$ Property 1 of Logarithms

$10^1 = x(x-3)$ Converted logarithm to exponential

$10 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5$ or $x = -2$

$x = 5$ checks, but $x = -2$ does not: $\log(-2) \stackrel{?}{=} 1 - \log(-2-3)$ means we would be taking a log of a negative number. So we have to reject -2 as a solution.

Therefore, the solution is $x = 5$.

Practice 2 $\log_3 x = 1 - \log_3(x+2)$

Did you get $x = 1$? Did you reject $x = -3$?

Example 3 $\log_6(2x-3) = \log_6 12 - \log_6 3$

Done using the method of Example 2:

$$\log_6(2x-3) - \log_6 12 + \log_6 3 = 0$$

$$\log_6\left[\frac{(2x-3)3}{12}\right] = 0$$

$$\log_6\left[\frac{2x-3}{4}\right] = 0$$

$$6^0 = \frac{2x-3}{4}$$

$$1 = \frac{2x-3}{4}$$

$$4 = 2x - 3$$

$$7 = 2x$$

$$x = \tfrac{7}{2}$$

Since all of the terms are logarithms, we can solve this in a different way: rewrite each side of the equation as a single logarithm. Since they have the same base, and logarithms are one-to-one, the expressions we are taking the logs of must be equal.

$$\log_6(2x-3) = \log_6\left[\frac{12}{3}\right]$$

$$\log_6(2x-3) = \log_6(4)$$

Thus, $2x - 3 = 4$, and then $x = \tfrac{7}{2}$.

Practice 3 $\log(x-4) - \log(3x-10) = \log\left(\frac{1}{x}\right)$

Did you get $x = 5$? Did you reject $x = 2$, since you'd be taking a log of a negative number?

Example 4 $\dfrac{\log_2(5x-6)}{\log_2 x} = 2$

The left side cannot be rewritten using properties of logarithms. But we can multiply both sides of the equation by the common denominator:

$$\log_2(5x-6) = 2\log_2 x$$

$$\log_2(5x-6) = \log_2 x^2 \quad \text{Property 3 of logarithms}$$

$$5x - 6 = x^2$$

$$0 = x^2 - 5x + 6$$

$$0 = (x-3)(x-2)$$

$$x = 3 \text{ or } x = 2 \quad \text{(and both work)}$$

Practice 4 $\dfrac{\log(8x-7)}{\log x} = 2$

Did you get $x = 7$? We have to reject $x = 1$ in this problem since $\log 1 = 0$, and no division by 0 is allowed!

Problems

1. $log_2\left(x^2-5x+14\right)=3$

2. $log_x 8=3$

3. $log_x 10=10$

4. $log\, x^2=log\, x$

5. $log\left(5x+1\right)=2+log\left(2x-3\right)$

6. $log_4\left(x+1\right)=2+log_4\left(3x-2\right)$

7. $log\left(\frac{4x+1}{2x+9}\right)=0$

8. $\frac{log\left(6x-8\right)}{log\, x}=2$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

____________Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

____________Student has mastered topic.